The Delivery and Control of Quality in Supplier-Producer Contracts

Diane J. Reyniers • Charles S. Tapiero
London School of Economics, London WC2A 2AE, United Kingdom
ESSEC 95021 Cergy Pontoise, France

We model the effect of contract parameters such as price rebates and after-sales warranty costs on the choice of quality by a supplier, the inspection policy of a producer, and the resulting end product quality. Both noncooperative and cooperative settings are explored. The paper's contribution is to highlight the importance of strategic and contractual issues in quality management.
(*Quality Control; Inspection, Game Theory*)

1. Introduction

The attempt to use improved quality to gain a competitive advantage has led firms to develop quality-sensitive industrial contracts. The quality of delivered materials and parts and its control through sampling or quality control procedures are thus important issues to reckon with in the negotiation of industrial contracts. Such contracts, between a Supplier of parts and a Producer of finished goods, may stipulate a rebate to be paid by the Supplier to the Producer for parts found defective upon inspection by the latter. In addition, the contract may stipulate how the warranty costs (paid to a downstream consumer of a faulty finished product) are to be divided between the Supplier and the Producer. One aim of this paper is to determine the effect of these contract parameters on the quality of the end product of the Supplier-Producer chain. We do this in a game theoretic context in which the Supplier may choose among manufacturing technologies with a cost-quality tradeoff, and the Producer may choose an inspection policy. Our main finding in this respect is that the end product quality resulting from the Supplier-Producer chain increases as the warranty costs are shifted from the Supplier to the Producer. More generally, we determine how the contract parameters and exogenous technology and inspection costs determine a unique equilibrium choice of technology by the Supplier and inspection policy by the Producer. Unlike most other research in the traditional SQC literature, this paper explicitly recognizes the potential conflicts between suppliers and producers and establishes a game theoretical framework in which to study these conflicts. The game theoretic approach to quality issues used in this paper was initiated by Reyniers (1992) in a context where technology was fixed and exogenous, but both parties devised their own sampling plans. Here we show how such a model can also be used to analyze technology choice.

We consider a model in which the Supplier of a part chooses a technology and this choice is not observed by the Producer, who independently decides on his inspection policy. There are many circumstances in which these assumptions are reasonable. As a generic example, consider a supplier of perishable goods who can influence the quality of these goods through his inventory policy. In particular, goods are of high quality if the supplier uses a FIFO (first in, first out) policy which ensures that products are rotated properly, and of bad quality if the supplier uses a (cheaper) LIFO (last in, first out) policy. The Producer who receives these goods cannot observe this choice of inventory policy.

Before describing our particular model and its analysis, we briefly describe the industrial problems which motivate our approach. The threat of incurring some of the costs associated with faulty parts is leading suppliers to invest in manufacturing technologies which generate

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improved quality and at the same time standardize the manufacturing process to be in accord with negotiated delivery contracts. Preferential suppliers of large firms are often determined on the basis of a consistent supply of quality. For the producer, in-house testing and quality management are needed to remove substandard lots from entering the manufacturing process, wasting both considerable machine and equipment time as well as labor costs. Various agreements have been reached in industry to circumvent difficulties caused by the delivery of defective parts and materials. For example, large firms such as Ford and GM may require that all sampling be performed by the supplier. Although such agreements lead to higher parts and materials prices, these added costs are considered worthwhile because of the savings reached by maintaining a smooth and defect-free supply.

Current practice in quality management and its control emphasizes the use of statistical control techniques (control charts, acceptance sampling, etc.) which seek to detect deviations from agreed on quality standards. These approaches, however, fail to recognize the increasingly intricate nature of manufacturing where uncertainties in the supply of materials may be subject not only to natural and random deviations but are also a function of the bilateral relationship between the supplier who manufactures the parts and the producer. A notable exception is Chew and Pisano (1990), where quality control is discussed in a contractual context. They suggest that as an alternative to vertical integration, longer term contracts with fewer suppliers may be required to improve quality.

In the application of Total Quality Control (TQC) some attempts are made to integrate quality control procedures into a broad management framework. These attempts, however, are not formalized and fail to recognize the complex motivations that underlie behavior of supplier and producer in a contractual environment. In practice, these issues are considered important, and clauses are inserted in supply contracts to provide incentives for the suppliers to comply with the terms of negotiated contracts. For example, some contracts stipulate that payments will be made after and as a function of the delivered quality. In other cases, there are warranties and agreements of various sorts which provide guarantees that some or all costs related to the delivery and use of substandard quality be sustained by the supplier.

Motivated by the above issues, we have developed a model by which to consider the effect of Supplier-Producer contracts on quality. We assume that a contract for the delivery of materials or parts has been negotiated and signed by a supplier and a producer. The contract stipulates penalties for defectives as follows. If a part is tested and found defective by the Producer, a rebate is paid by the supplier which in effect reduces the price of the part to the producer. The supplier incurs a repair cost and the producer is supplied with a nondefective unit. If a defective part enters the manufacturing process undetected, it will end as a finished product sold to a consumer who will detect the defective part for sure. When this occurs, manufacturing and post-sales costs are incurred which are shared between the producer and the supplier according to the contractual agreement. For simplicity, we assume that the supplier and the producer are risk neutral and fully informed of each other's objectives and manufacturing potential. We study the effects of the contract parameters on the propensity of the supplier to deliver good quality and on the propensity of the producer to inspect incoming parts. To this end, we formulate a two-person nonzero sum game (e.g. Owen (1982), Thomas (1986)). The supplier has a finite number of alternatives for delivering quality (for simplicity we consider two alternatives only). Practically, these alternatives are defined by combined manufacturing technologies, quality management efforts, etc. whose outcome provides a product and a distribution over the yield of defectives delivered. The producer has two alternatives: inspect an incoming product or not.

We determine unique Nash equilibria as a function of the exogenous costs, technology properties, and the contract parameters. We find that the type of equilibrium (e.g. the supplier delivers high quality and the producer inspects incoming lots) depends on the inspection costs and the ratio $\Delta T / \Delta p$, where ΔT is the incremental cost of the better technology and Δp is the incremental probability of a defective part using the inferior technology. Intermediate values of these parameters result in a unique mixed strategy equilibrium. Our main findings are that at the noncooperative equilibrium,

1. The probability that the Producer inspects is increasing in $\Delta T / \Delta p$

2. The probability of using inferior technology is increasing in the Producer's inspection cost.

3. The final quality of the Supplier-Producer chain is a decreasing function of the proportion of the warranty cost borne by the Supplier, a decreasing function of the Producer's inspection cost, and an increasing function of the ratio $\Delta T / \Delta p$

Note that in the above equilibrium analysis, it is assumed that the choice of technology by the Supplier and the choice of inspection policy by the Producer are not specified in the contract, but are rather decision variables in the post-contract game. If we consider the possibility that these choices are determined by a binding agreement in a subsequent contract, then we are led to consider the bargaining process that results in such a second contract. We take a Nash Bargaining approach to this cooperative problem. We determine the Nash Bargaining Solution as a function of the parameters of the first contract (rebate and division of warranty costs). Calling the increased payoff obtained in the Nash Bargaining Solution relative to the noncooperative equilibrium the "value of cooperation," we find that

1. The value of cooperation to both players is a decreasing function of the rebate paid by the Supplier to the Producer for each defective part found by the latter.

2. The value of cooperation to both players is an increasing function of the proportion of the warranty costs paid by the Producer.

2. A Noncooperative Quality Game

Consider a supplier of parts whose quality production potential is defined by two technologies $i = 1, 2$ with respective probabilities p_i of a defective part. We assume throughout the paper that $p_1 > p_2$ so that p_1 corresponds to the production of poor quality and p_2 to the production of high quality. For alternative i, the unit cost of production borne by the supplier is T_i where $T_1 < T_2$. When the producer receives a lot, at price π per unit, he chooses whether or not to test it. If the lot is tested, a cost m per unit is incurred by the producer and the outcome observed. If a unit is defective, it is repaired by the supplier who incurs a repair cost C and the price of the unit is reduced by $\Delta\pi$, $\Delta\pi \geq 0$. This rebate can be interpreted as a transfer from the supplier to the producer to provide an incentive for the supplier to deliver good quality products. If the lot is not tested by the producer and a part is sold defectively, then its post-sales failure cost R is shared by the supplier and the producer according to a sharing rule on which they agreed at the time the contract was signed. We define this sharing rule by a parameter α such that $(1 - \alpha)R$ will be borne by the producer and αR by the supplier. We assume that $\Delta\pi + C > \alpha R$ i.e., the cost to the supplier of a defective part found on inspection is larger than the cost of a defective part detected after sale because the latter cost is shared with the producer.

The game described above corresponds to a bimatrix (A, B) with entries (a_{ij}, b_{ij}), $i = 1, 2$; $j = 1, 2$, the expected payoff to the producer and the supplier respectively. Let $j = 1$ denote the producer's decision to test the incoming part and $j = 2$ its alternative, not to test it. The supplier's decision to deliver low quality corresponds to $i = 1$ and high quality corresponds to $i = 2$. In addition, assume that θ is the producer's selling profit (net of manufacturing costs) $\theta > \pi$. For given i and a risk neutral producer, the expected payoff will be.

$$j = 1(\text{Test}). \quad a_{i1} = \theta - m \quad [\pi - p_i \Delta\pi], \quad i = 1, 2$$

$$j = 2(\text{No Test}). \quad a_{i2} = \theta - [\pi + p_i(1 - \alpha)R], \quad i = 1, 2,$$

where m is the cost of testing an incoming part borne by the producer. Similarly, the expected payoffs realized by the supplier are.

$$b_{i1} = [\pi - p_i(\Delta\pi + C) - T_i] \quad \text{and}$$

$$b_{i2} = [\pi - p_i \alpha R - T_i], \quad i = 1, 2.$$

We will now look for Nash equilibria of the bimatrix game above. These equilibria determine both the producer's sampling policy and the quality delivered by the supplier. By definition of a Nash equilibrium, if the Nash equilibrium is unique and both the producer and the supplier adopt their Nash equilibrium strategy, they will have no incentive to deviate from it. The following proposition presents the Nash equilibria in terms of the contract terms agreed on by the supplier and the producer

PROPOSITION 1. *Consider the bimatrix game* (A, B) *and assume* $\Delta\pi + C > \alpha R$. *For any given set of parameters there is a unique Nash equilibrium as indicated in Table 1 below, where* $\mu = p_1(\Delta\pi + (1-\alpha)R)$, $\nu = p_2(\Delta\pi + (1-\alpha)R)$, $\Delta p = p_1 - p_2$, $\Delta T = T_2 - T_1$. *The equilibrium is in pure strategies except when* $\alpha R < \Delta T/\Delta p \le \Delta\pi + C$ *and* $\nu < m \le \mu$, *in which case there is a mixed strategy Nash equilibrium with probability of inspection*

$$q^* = [\Delta T/\Delta p - \alpha R]/(\Delta\pi + C - \alpha R)$$

and probability of low quality $x^* = [m - \nu]/[\mu - \nu]$

PROOF. To determine the Nash equilibria, consider the expected payoffs for each of the players. For the supplier, the expected payoff is given by:

$$\begin{aligned} v(q, x) = {} & [\pi - p_1(\Delta\pi + C) - T_1]xq \\ & + [\pi - p_1\alpha R - T_1]x(1-q) \\ & + [\pi - p_2(\Delta\pi + C) - T_2](1-x)q \\ & + [\pi - p_2\alpha R - T_2](1-x)(1-q) \end{aligned}$$

For the producer, the expected payoff is given by:

$$\begin{aligned} u(q, x) = {} & (\theta - m - [\pi - p_1\Delta\pi])xq \\ & + (\theta - [\pi + p_1(1-\alpha)R])x(1-q) \\ & + (\theta - m - [\pi - p_2\Delta\pi])(1-x)q \\ & + (\theta - [\pi + p_2(1-\alpha)R])(1-x)(1-q) \end{aligned}$$

To find reaction functions we optimize $v(q, x)$ and $u(q, x)$ w.r.t. x and q respectively. The reaction functions are shown in Figure 1. For a given producer inspection policy q, the supplier will set x, the quality policy, as follows: $x = 1$ if $q < q^*$, $x = 0$ if $q > q^*$ and $x =$ any $x \in [0, 1]$ if $q = q^*$. For a given quality policy x, the producer will set the inspection probability q as follows: $q = 1$ if $x > x^*$, $q = 0$, if $x < x^*$ and $q =$ any $q \in [0, 1]$ if $x = x^*$. The Nash equilibrium occurs where the reaction curves intersect. □

Figure 1 The Unique Nash Equilibrium

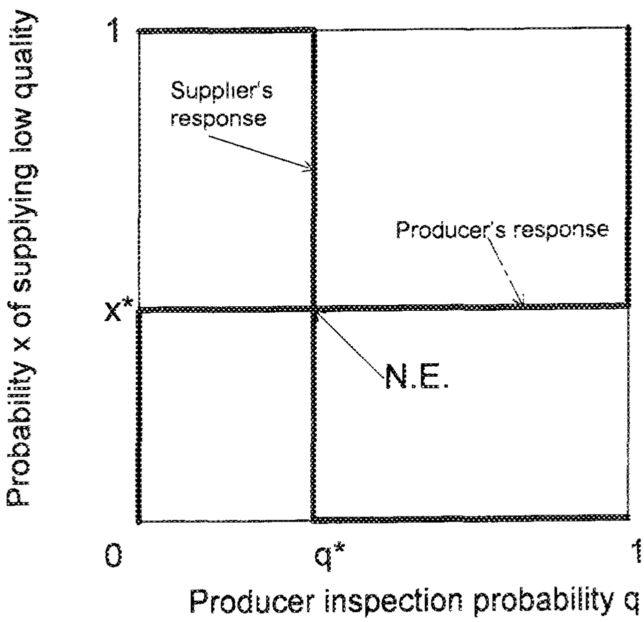


Table 1 Type of Nash Equilibrium in Noncooperative Quality Game

	$m \le \nu$	$\nu < m \le \mu$	$m > \mu$
$\Delta T/\Delta p < \alpha R$	High Quality, Inspection	High Quality, No Inspection	High Quality, No Inspection
$\alpha R < \Delta T/\Delta p < \Delta\pi + C$	High Quality, Inspection	Mixed, Mixed	Low Quality, No Inspection
$\Delta T/\Delta p > \Delta\pi + C$	Low Quality, Inspection	Low Quality, Inspection	Low Quality, No Inspection

Using Proposition 1, we can identify Nash equilibria for any given set of parameter values. In particular, Table 1 shows our results in terms of the relative production cost differential $\Delta T/\Delta p$, and the inspection cost m. If the relative production cost differential is small then the supplier will always provide high quality. If the production cost differential is large then the supplier will always provide low quality. If the inspection cost is relatively small then it will be optimal for the producer to inspect incoming lots. If the inspection cost is relatively large it is optimal for the producer not to inspect at all. Only when the relative production cost differential and/or the inspection cost are neither small nor large do we get more interesting results. For intermediate values of $\Delta T/\Delta p$ the supplier will supply high quality if m is small (and hence there will be an inspection) and low quality if m is large (when there is no inspection). Similarly, for intermediate values of m, the producer does not inspect when $\Delta T/\Delta p$ is small (because the supplier will deliver high quality) and he will inspect when $\Delta T / \Delta p$ is large (the supplier delivers poor quality). When both parameters $\Delta T / \Delta p$ and m take intermediate values, we find a unique Nash equilibrium in mixed strategies At this equilibrium, the probability of inspection q^* is a linear function of $\Delta T / \Delta p$ varying from 0 to 1 as $\Delta T / \Delta p$ ranges from αR to $\Delta\pi + C$ At the same time the probability of delivering low quality x^* is a linear function of the inspection cost m, varying from 0 to 1 as the inspection cost ranges from ν to μ In other words, the probability of inspection increases with the relative cost of producing good quality and the probability of low quality increases with the inspection cost

We can provide a physical and real interpretation of these randomized strategies corresponding to the quality supplied by the supplier and the inspection policy of the producer If $i = 2$ corresponds to the production process being in control which implies a special effort compared to $i = 1$ (the production process is out of control), with $p_1 > p_2$, then the magnitude of this effort could determine x The larger the effort, the smaller the probability x and vice versa. The producer's inspection policy at a mixed strategy equilibrium can be interpreted as the producer inspecting a fraction q of incoming lots

It is instructive to see how the equilibrium strategies depend on the contract parameters α and $\Delta\pi$. From the results in Proposition 1 it is easy to verify that the probability of inspection is decreasing in the rebate $\Delta\pi$ and in the supplier's share α of the post-sales costs In other words, if the contract parameters are set sufficiently high the supplier faces a lower inspection probability at equilibrium Similarly, the probability of delivering low quality is increasing in the producer's share of post-sales cost (or decreasing in the supplier's share) and decreasing in the rebate. This means that, if the supplier's penalties $\Delta\pi$ and α are set sufficiently high, he will be likely to provide high quality at equilibrium

We now consider how the final quality of the end product of the Supplier-Producer chain depends on the contract parameters α and $\Delta\pi$ At the unique equilibrium resulting from these parameters we look at the probability that a final product is defective This obviously depends directly on the technology chosen by the Supplier and the inspection policy of the Producer, and only indirectly on the contract parameters. We begin by observing that

$$\Pr(\text{final good is defective}) = \begin{cases} 0 & \text{if inspected} \\ p_1 \text{ or } p_2 & \text{if not inspected} \end{cases} \quad \text{and so}$$

$$\begin{aligned} \Pr(\text{final good is defective}) &= p_1 x(1-q) + p_2(1-x)(1-q) \\ &= (1-q)(xp_1 + (1-x)p_2) \\ &= \frac{m(\Delta\pi + C - \Delta T/\Delta p)}{(\Delta\pi + C - \alpha R)(\Delta\pi + (1-\alpha)R)} \end{aligned}$$

From the penultimate equation above we see that the probability of a defective final good is decreasing in the inspection probability q and increasing in the probability of delivery of low quality by the supplier From the final equation it can be shown that the probability of a defective final good is

- increasing in inspection cost (This is what one would expect)
- decreasing in the ratio $\Delta T / \Delta p$. (This is counterintuitive The reason is that when the production cost differential $\Delta T / \Delta p$ is large, inspection is more likely.)
- increasing in α, the supplier's share of post-sales cost (When α is large, the Producer is not likely to inspect.)
- increasing in the supplier rebate $\Delta\pi$, for small values of $\Delta\pi$, and decreasing in $\Delta\pi$, for large enough values of $\Delta\pi$

The implications of these results are of course numerous The supply of quality and its control in a conflicting environment depend on the contract parameters $\Delta\pi$ and α In this sense, contract parameters affect post contract behavior and can provide an incentive for the supply of high or low quality The implications of this statement for contract design and management are obvious The behavioral effects of quality contracts cannot be neglected in the design of contracts That is, in the negotiation and the selection of rebate parameters $\Delta\pi$ and α, it should be clear that the subsequent successful implementation of the contract will depend on the economic effects of the contract on each of the parties' payoffs Our analysis sets up the quality control problem into an appropriate framework for a conflicting environment. Quality control and sampling are used not only to assure the producer of the incoming product quality but also as a "threat" against the supplier if he delivers defective parts. That is, the role and importance of quality control may in fact be far broader than just product assurance as currently presumed. Tapiero (1987) has already shown that quality control may be used to learn more about the production process. Here we show how it can be used to manage compliance of suppliers to the terms of contracts.

3. A Cooperative Quality Game

In most industrial situations, mutual interests by producers and suppliers lead to cooperation in delivering quality products. In this section, we consider a cooperative game between the supplier and the producer and we assess the value of cooperating. Assume again that payoffs are given by the bimatrix (A, B). The solution concept we use is the Nash Minimax Bargaining solution. This solution concept has some desirable properties and is a rational procedure for settling bargaining problems [see e.g. Thomas (1986)].

Due to the tedious computations required to obtain analytical results, we will only consider the realistic parameter set corresponding to the case of a unique Nash equilibrium in mixed strategies as stated in Proposition 1. The other cases can be dealt with similarly. The following proposition presents the Nash Minimax Bargaining solution in terms of the contract parameters.

PROPOSITION 2. *Consider the producer-supplier game defined by the bimatrix (A, B). Assume $\nu < m < \mu$, and $\alpha R < \Delta T/\Delta p < \Delta\pi + C$. In addition, let*

$$m > p_1\Delta\pi + p_2(1-\alpha)R$$

and define

$$u^* = \theta - \pi - [(1-\alpha)mR] / [2(\Delta\pi + (1-\alpha)R)] \quad \text{and}$$

$$v^* = \pi - m[\alpha R - \Delta T/\Delta p]/[2(\Delta\pi + (1-\alpha)R)] - [p_1T_2 - p_2T_1]/\Delta p$$

Then the Nash minimax bargaining solution is given by $u_B = u^*$, $v_B = v^*$, when $m > 2\nu$ and $u_I = \theta - \pi(1-\alpha)p_2R$, $v_B = \pi - \alpha p_2 R - T_2$, otherwise.

Further, let u_0 and v_0 be the security values to both players (which are in this case identical to the Nash equilibrium payoffs):

$$u_0 = (\theta\pi) - m(1-\alpha)R/[\Delta\pi + (1-\alpha)R] \quad \text{and}$$

$$v_0 = \pi(p_1T_2 - p_2T_1)/\Delta p.$$

Then the value of cooperation to the producer and the supplier are given by

$$\Delta u = u_B - u_0 = [(1-\alpha)mR]/[2(\Delta\pi + (1-\alpha)R)],$$

$$\Delta v = v_B - v_0 = [m(\Delta T/\Delta p - \alpha R)] / [2(\Delta\pi + (1-\alpha)R)] \quad \text{for } m > 2\nu; \quad \text{and}$$

$$\Delta u = u_r - u_0 = [(1-\alpha)R(m-\nu)]/[\Delta\pi + (1-\alpha)R],$$

$$\Delta v = v_B - v_0 = p_2(\Delta T/\Delta p - \alpha R) \quad \text{otherwise.}$$

Hence, both the supplier and the producer have an incentive to cooperate.

PROOF. Consider the bimatrix game (A, B) defined in Proposition 1. Since $p_2 < p_1$ we have $a_{11} > a_{21}$ and $a_{12} < a_{22}$. Since $\nu < m < \mu$ we also have $a_{11} > a_{12}$ and $a_{21} < a_{22}$. Further, due to the inequality $\alpha R < \Delta T/\Delta p < \Delta\pi + C$, it follows that $b_{11} < b_{21}$ and $b_{12} > b_{22}$. Finally, from $\alpha R < \Delta\pi + C$ we have $b_{11} < b_{12}$ and $b_{21} < b_{22}$. These relationships imply that $b_{11} < b_{21} < b_{22} < b_{12}$. If we further restrict parameter values to $m > p_1\Delta\pi + (1-\alpha)p_2R$ and hence $a_{11} < a_{22}$, the payoff region can be drawn as shown in Figure 2(a)–(b). As a result the set of Pareto optimal payoffs (the negotiation set) consists of the line segment (a_{12}, b_{12}) to (a_{22}, b_{22}).

In order to find the maximin bargaining solution we first calculate the maximin values for both players as the status quo point (Thomas (1986)):

$$u_0 = [a_{11}a_{22} - a_{12}a_{21}]/[a_{11} + a_{22} - a_{21} - a_{12}],$$

$$v_0 = [b_{11}b_{22} - b_{12}b_{21}]/[b_{11} + b_{22} - b_{21} - b_{12}]$$

When we insert the entries (a_{ij}, b_{ij}) as defined above, we obtain (u_0, v_0) as stated in the proposition. To find the Nash bargaining solution using these values as status quo point, we maximize $(u - u_0)(v - v_0)$ subject to (u, v) in the negotiation set, i.e. for

$$(u - a_{12})(b_{22} - b_{12}) = (v - b_{12})(a_{22} - a_{12})$$

Figure 2a The Negotiation Set: Player 2

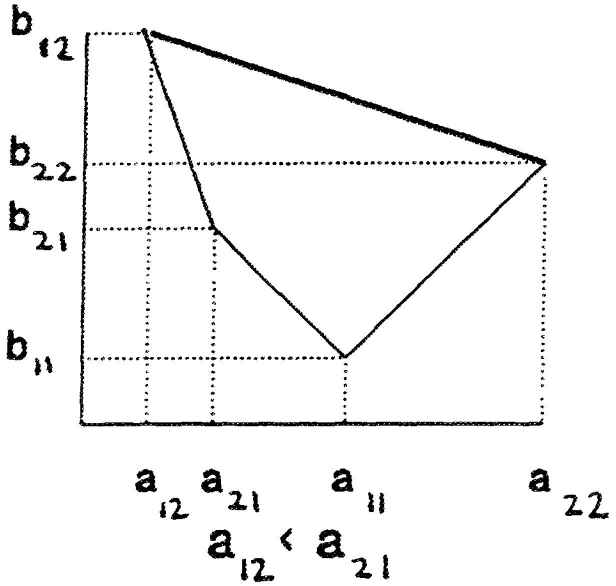
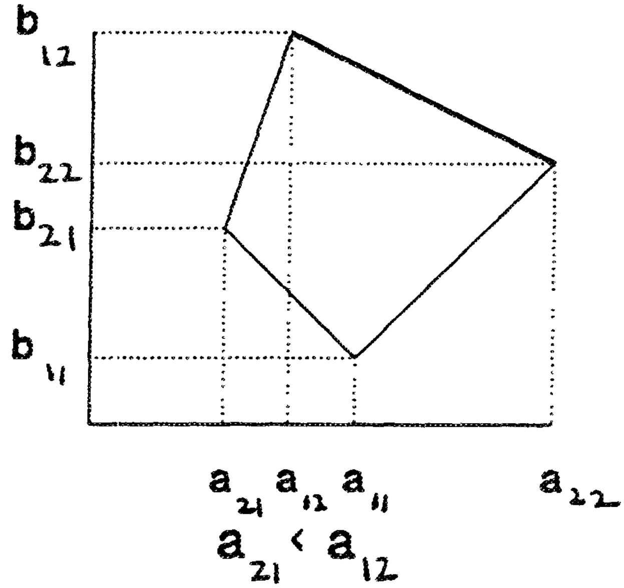


Figure 2b The Negotiation Set Player 1



This maximization leads to u^* and v^* as in the proposition. However, (u^*, v^*) is the Nash Bargaining Solution only if it is on the line segment between (a_{12}, b_{12}) and (a_{22}, b_{22}). Hence,

$$u_B = \min(\max(a_{12}, u^*), a_{22}) \quad \text{and}$$

$$v_B = \max(\min(b_{12}, v^*), b_{22})$$

Since $a_{12} < u^*$ for $m < 2\mu$ this reduces to $u_B = \min(u^*, a_{22})$ and $v_B = \max(v^*, b_{22})$ or $u_B = u^*$, $v_B = v^*$ when $u^* < a_{22}$ or $m > 2\nu$, else $u_B = a_{22}$, $v_B = b_{22}$ as stated in the proposition. Finally, the difference between the Nash bargaining solution and the security solution is by definition the potential gain that the supplier and the producer can obtain by cooperating, which is defined in the proposition by Δu and Δv. □

From Proposition 2 we know that since $\Delta u \geq 0$ and $\Delta v \geq 0$ it is always worthwhile for the producer and the supplier to cooperate. Define $\phi = \Delta\pi/((1-\alpha)R)$. Then $\Delta u = m/[2(1+\phi)]$ for $m > 2\nu$ and $\Delta u = (m-\nu)/(1+\phi)$ for $m < 2\nu$. Hence, $\partial\Delta u/\partial\phi < 0$ and therefore, the value of cooperation for the producer decreases when $\Delta\pi$ and/or α increase(s). This is plausible when these contract parameters increase, the cost of defectives supply is shifted to the supplier. As a result, the value of cooperation to the producer decreases.

The supplier's expected payoff improvement through cooperation can be examined as follows: $\partial\Delta v/\partial\alpha \geq 0$ for $m > 2\nu$ only when $\Delta T/\Delta p \geq R + \Delta\pi$. Given the assumptions of the proposition this could occur only when $R < C$ which in practice is very unlikely. When $m \leq 2\nu$ we find $\partial\Delta v/\partial\alpha < 0$. Therefore the incentive for the supplier to cooperate decreases with α, his share of the after sales replacement cost. Also $\partial\Delta v/\partial\Delta\pi < 0$ when $m > 2\nu$ and $\partial\Delta v/\partial\Delta\pi = 0$ when $m \leq 2\nu$. That is, when $\Delta\pi$ increases there is less of an incentive to cooperate. This is due to the fact that when inspection costs are large, this can be exploited by the supplier in a non-cooperative setting.

Finally, note that $\partial\Delta v/\partial(\Delta T/\Delta p) \geq 0$. That is, when the relative production cost differential increases there is more of an incentive for the supplier to cooperate. Although we find that for the conditions on the parameters in Proposition 2 both parties gain from cooperation, in general cooperation does not always pay for both parties since the Nash bargaining solution payoff is not always better than the Nash equilibrium payoff

4. Numerical Example

The results derived earlier can be analyzed numerically. For our current purposes assume the following parameters: $\Delta\pi = 1$, $\alpha = 0.3$, $R = 1.5$, $m = 0.25$, $p_1 = 0.15$, $p_2 = 0.05$, $T_2 = 1$, $T = 0.95$, $k = 10$, $\pi = 7$, $C = 0$. The bimatrix (A, B) is then given by

$$(A, B) = \begin{bmatrix} (2.9, 5.9) & (2.8425, 5.9825) \\ (2.8, 5.95) & (2.9475, 5.9775) \end{bmatrix}$$

The cooperative payoff region of all payoff pairs in the bimatrix is shown in Figure 3. The set of Pareto efficient payoff pairs is given by the line segment joining (a_{12}, b_{12}) and (a_{22}, b_{22}).

The Nash equilibrium is given by $x^* = 0.7195121$ and $q^* = 0.090909$, i.e., the probability of poor quality is 0.72 and the probability of inspection is 0.091. The security values are $u_0 = 2.872$ and $v_0 = 5.975$ so that $(u_B, v_B) = (2.936, 5.978)$ becomes the Nash bargaining solution. The value of cooperation for the producer is $2.936 - 2.872 = 0.64$ while for the supplier it is $5.977 - 5.975 = 0.002$. This is illustrated in Figure 3.

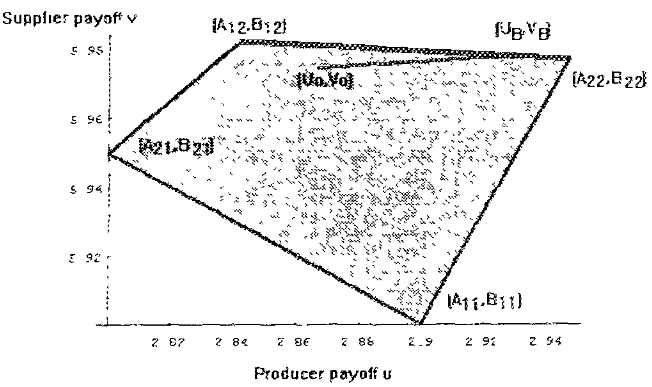
5. Discussion

Quality provision and inspection policies depend on the nature of the industrial contract negotiated between the supplier and the producer. Once such a contract is agreed, there is an inherent conflict in which each party wants the other to bear the cost of producing a high quality product: the Producer wants the Supplier to use a costly but high quality technology, and the Supplier wants the Producer to ensure quality through the adoption of an effective but high cost inspection policy. Recognizing this conflict, we have developed a model in which to investigate its possible resolution. We have applied both non-cooperative and cooperative game theoretic notions to this model. We have determined how the solutions thus obtained, as well as the final quality of the end product, depend on the physical and contractual parameters of the model. In particular we have given explicit formulae, in terms of these parameters, for the probability that the supplier will adopt a high quality technology and for the probability that the producer will decide to inspect the output of this technology. In addition we have derived the formula for the final quality (the probability that the final product is not defective). Many qualitative observations can be made on the basis of these results. For example, final quality of the end product will be increased if the supplier's share of the post-sale warranty costs is decreased and the incentive for post-contract cooperation is increased if the rebate contracted for the supplier to pay the producer for each part the latter finds defective is increased.

Although we have restricted our attention to all-or-nothing inspection, the extension to various sorts of inspection schemes (see e.g. Duncan 1974) is straightforward. Technically, this is equivalent to increasing the number of alternatives faced by the producer. Thus, rather than having a strategy space consisting of testing or not testing we could augment the strategy space to account for any number of inspection alternatives. The solution to this problem, (unlike the classical approach to inspection and quality control which maintains the same inspection procedure), could result in a mixture of inspection procedures (i.e., a mixture of intensive and superficial inspection procedures).

Further, considering the dynamic nature of the supplier-producer relationship, we could model inspection procedures which change from time to time as a function of the experience gained through inspection and repeating the game. Although such problems might be difficult to solve analytically, solutions can be found quantitatively (once the concept of solution has been clearly stated). Over time, the decision to inspect is a function of the producer's "belief" regarding the sup-

Figure 3 Payoff Region and Nash Bargaining Solution



plier's quality Information about the product s quality arises due to failure of past sales (which have returned as defective products) and inspection sampling performed by the producer. In other words, the prior information about product quality, appropriately updated from period to period, may be used to determine when and how much to inspect In this sense, inspection policies in supplier-producer contracts should be modeled as multi-stage games

Other issues such as the structure of industrial markets (e g competitive markets, franchises with lock in contracts, etc) affect the quality delivered and the inspection procedures pursued In lock-in contracts for example, there is a potential for opportunistic behavior by the franchiser or by the franchisee which can be partially moderated if inspection procedures are introduced as an integral part of the commercial contract which regulates exchange and profits sharing Thus, in a large number of contractual agreements where outcomes are imperfectly observed and/or there may be some problems in monitoring behavior, the separation of the contract terms, which provide the incentive structures, and players' optimal actions can be misleading To solve these problems, which often occur in practice, a broader framework of analysis, sensitive to the inherent conflict in contractual agreements is needed [1]

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